Topic Study Group 10

Teaching and Learning of Measurement

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ABSTRACT This chapter presents the aims, the work done during the online conference, and perspectives of TSG-10 — teaching and learning of measurement.

Keywords: Teaching and learning of measurement; Variability of phenomena; School arithmetic; Multiplicative reasoning; Interplay between space and measure; Interdisciplinary practices.

1. Aims of the TSG

Measurement topics in TSG-10 include typical domains such as length, area, angle, volume, and mass but also those less studied, such as time, and those commonly visited in science and engineering education. Overall, internationally, there seems to be a lack of attention to measurement instruction in mathematics education, especially at the primary levels. This is in spite of measure's links to everyday contexts and to STEM disciplines. Although the historic role of measurement has declined in some areas of mathematics, substantive mathematical ideas, such as number and quantity, originated in practices of measure, and these origins continue to be important for student learning about these ideas.

The main objective of the TSG is to better understand the conditions and constraints on teaching and learning measurement in international contexts (from primary to university levels) and to consider new approaches to learning both of measurement and of related forms of mathematics. A diversity of perspectives was expected, e.g., theoretical, methodological, historical, epistemological or empirical, and from various points of view, including teachers' practices, students' learning, as a mathematical subject, teacher education, curriculum, and so on.

1.1. Submissions

There were 18 submissions in the TSG. These submissions were from a diversity of locales (North America: 3; Asia: 5; Europe: 4; Africa: 3; Australia: 2; Eurasia: 1).

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Thirteen oral presentations and one poster were planned to be presented in Shanghai during the online conference (but eventually only twelve orals presented).

1.2. Sessions

The TSG work was organized in three sessions: 90 minutes, 90 minutes, and 120 minutes. A brief presentation by the chair (Christine Chambris) introduced the work of the three sessions. She presented the different themes. Then, each presentation was followed by time for discussion. Last, Rich Lehrer drew perspectives for the TSG, at the end.

1.3. Paper list

Tab. 1. List of papers presented

Paper and author(s)	
[1]	Rethinking measure. *Petronilla Bonissoni, Marina Cazzola, Gianstefano Riva,* ***Ernesto Rottoli****, and Sonia Sorgato* (Italy).
[2]	Can length measurement estimation activities contribute to learners' improvement on number line estimation tasks? ***Pamela Vale*** (South Africa).
[3]	Measurement units and numeration units: What reveals the introduction of a "mixed" table in decimals teaching. *Christine Chambris,* ***Lalina Coulange****, and Grégory Train* (France).
[4]	Role of "error" in teaching-learning measurement. ***Ishan Santra*** *and Jeenath Rahaman* (India).
[5]	An investigation of teachers' explanatory talk when introducing standard units of measuring length to standard 4 learners in Malawi. *Liveness Mwale* (Malawi). (Due to connection issues this paper was unfortunately not presented)
[6]	Insight into pupils' errors in solving problems involving calendar dates through analysis of knowledge states. ***Phei Ling Tan*** *and Liew Kee Kor* (Malaysia).
[7]	Measuring the teacher's arm span: Interpreting a data modeling sequence through an aesthetic lens. ***Russell Tytler****, Peta White, and Joseph Ferguson* (Australia).
[8]	The use of geometric construction problems to solve measurement problems at middle school. ***Gbaguidi Ahonankpon Florent*** (Benin).
[9]	Conceiving volume as a multiplication of three quantities: the cases of Stan and Sloane. ***Samet Okumus*** (Turkey).
[10]	Articulations between mathematics and physics education: the concept(s) of unit of measurement, from geometry to formulas. ***Charlotte de Varent*** (France).
[11]	Dynamic measurement for area and volume. ***Nicole Panorkou*** (USA).
[12]	Teaching with clocks: Instrumental dynamics' effects on time learning. ***Chaereen Han*** *and Oh Nam Kwon* (South Korea).
[13]	Young students learning the mathematics of measurement through an interdisciplinary approach. ***Peta White****, Russell Tytler, Joanne Mulligan, and Melinda Kirk* (Australia).
[14]	An explorative study of using Picture books to support students' learning of measurement in primary education. *Lanjie Sun* (China). (Poster. Not presented during TSG sessions)

2. Conference Themes

Among the diversity of the topics in the papers to be presented, four main themes emerged that structured the sessions: relationships between numbers and units,

negotiating measure and its meanings, interplay between conceptions of space and measure, and fostering development of measure through interdisciplinary practice.

In the first session, papers focused on relationships between numbers and units. For decades now, it is acknowledged in mathematics education that contrary to academic number sets constructions, teaching fractions and decimals should be based on measurement. That said, the specific roles that measurement units might play in teaching and learning mathematical concepts such as these were considered.

Rottoli and colleagues[1] discussed a perspective that views natural number as generated by counting and fraction as generated by comparison. In this perspective, the unit is a common unit between the two compared quantities obtained through subtractions. Vale[2] suggested that through preparing sticks of relative sizes to a referent (i.e., subunits), students develop knowledge in estimation on the number line. Chambris et al.[3] highlighted that poor understanding of units of measurement and of numeration impedes students' abilities to relate place value to metric units. They suggested that a multiplicative understanding of units is required for making this bridge.

The second session consisted of empirical studies of students and teachers that were negotiating measure and its meanings. Observing students and teacher's prompts, Santra and Rahaman[4] demonstrated missed and successful opportunities for "errors" to be a resource in teaching and learning of measurement, and specifically in stressing the variability of phenomena. Tytler and colleagues[7] collected a series of data involving grade-4 students measuring their teacher's arm-span and then constructing representations of the collection of measures. Analyses indicated how students' aesthetics of data display guided how they constructed their representations, and also how their teacher elicited these aesthetic judgments to help students understand some of the common conventions employed to visualize data. Tan and Kor[6] focused on students' errors in calendar learning, and this led to a discussion of relevant tasks to assess student knowledge about the measurement of time, including students' understandings of the meaning of time.

The third session focused on interplay between conceptions of space and measure. Florent[8] presented three geometrical problems that aimed to involve students in the construction of a measured quantity. Such problems seem to be promising for fostering reasoning in geometry to expand the meaning of a geometrical quantity. Okumus[9] investigated the students' understandings of the meaning of multiplication in the volume formula of a rectangular prism. Despite forming stacks of triangles, students were not able to relate the area of a triangle and the height of the stack to the measure of the volume of the prism. Panorkou[11] studied students' development in measurement reasoning mediated with dynamic geometry. She captured instances of meaning students generated by manipulating the software. Students seemed to perceive how dragging a surface area through a length generates volume. They also were able to identify units of area formed through dragging a certain distance a line of a given length, similarly for units of volume formed through dragging a surface of a given area through a particular distance. Students linked the latter to a volume formula (base × height) for a prism. Last, students seemed to reason in terms of continuous change: to

make the volume *n* times bigger, they need to make the area of the base *n* times bigger. This also demonstrates co-variational reasoning in students. Opportunities for students to reason dynamically about area and volume could be very productive but are not yet incorporated into school curriculum. For instance, in a study of textbooks commonly used in France to teach concepts of measure, de Varent[10] found that length units and area units were related in only one of 13 textbooks in France, in the setting of the formula of the area rectangle. This questions not only coherence between geometry and arithmetic, but also how co-variation between the different quantities is taught. Regarding relationships between time and space, Han and Kwon[12] demonstrated how specific use of two kinds of educative devices — clocks with linked and independent hands — depending on teachers' competences in Korea, engaged students in co-variational reasoning or not.

Finally, the last theme of the TSG was about fostering development of measure through interdisciplinary practice. White and colleagues[13] gave an inspiring example of how interdisciplinary practice supported grade-1-and-2 students to learn robust mathematics of measurement. Based on ecological inquiry of their surroundings, students developed means of measuring, recording, and constructing a display of a class collection of measures within a pedagogical context that notably encouraged exploration of ideas and constructing consensus.

3. Summary, Future Directions and Suggestions

The fundamental problematic of measurement can be seen as inventing symbolic means for describing and quantifying attributes of objects and of quantitative relations among them. The relationship between measure of an attribute and understandings of the nature of the attribute are co-constituted. Thus, communities render measures and phenomena reproducible, so that individual acts of measure are guided and enlarged by a community that values reproduction and clarity about acts of measure. Expressions of and work with measures require development of means, notably more elaborate systems of units and of numbers.

Beyond these epistemological considerations, major teaching and learning issues in measurement appear. Reproducibility of phenomena raises questions about variability: Why are our measures varying? Variability expands the reach of measure, notably in terms of characterizing its sources, of reasoning about co-variation, and of important ideas like distribution. Numbers emerge from the need to record relations through comparison processes that involve units (i.e., a given attribute that can at time be chosen arbitrarily) that enable records of phenomena. Grounding measure in space both via construction, and symbolic means reflect interaction between structures of phenomena and measures. Yet, though promising experiences, analyses of actual learning or teaching process and resources reveal the need of continuing efforts in the domain.

This leads to future directions for research. Interdisciplinary practices appear to be a powerful means to develop meaningful practice of measure, both for students and

teachers. Further research is needed that clarifies conditions of teaching and learning of practices and concepts of measure that foster deeper understanding of arithmetic, including multiplicative reasoning, and co-variational reasoning. There is also a critical need to develop longitudinal studies in order to better understand changes in students, and teachers. An emphasis on students' changing conceptions of measure suggests the need to develop means and tools for broader assessment, both for concepts to be learnt and teaching practices.

Generally speaking, TSG-10 had regular attendants who were ready to engage in rich discussion throughout the three sessions in a receptive atmosphere. The relatively small number of papers confirms that there is a lack of attention to this domain. Despite this, new issues in teaching and learning were raised, and the necessity for further international studies in the domain of measurement was reaffirmed. We hope that the topic study group dealing with measurement continues as a well-recognized group of the congress.